MATHEMATICS METHODS
UNITS 3 AND 4
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(5 marks)**

The value, V, of a school photocopier is given by $V = 1350e^{-0.22t}$, where t is the time in years since the copier was bought.

(a) Calculate the value of the photocopier after 8 years. (1 mark)

$$V(8) = \$232.26$$

(b) The rate of change of value of the photocopier can be written as $\dfrac{dV}{dt} = aV$. State the value of the constant a. (1 mark)

$$a = -0.22$$

(c) At the same time the photocopier was purchased, the school also bought a computer for $2350. One year later it was valued at $1688. The value of this computer after t years is given by $2350e^{-kt}$, where k is a positive constant.

   (i) Determine the value of k. (1 mark)

$$1688 = 2350e^{-k(1)}$$

$$k = 0.331$$

   (ii) After how long did the value of the computer first fall below that of the photocopier? (2 marks)

$$2350e^{-0.331t} = 1350e^{-0.22t}$$

$$t = 4.99$$
$$\approx 5 \text{ years}$$

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Question 10 **(9 marks)**

(a) The discrete random variable X has the probability distribution shown below.

x	0	2	4	6	8	10
$P(X = x)$	0.1	$5a$	0.25	0.2	$3a$	a

(i) Determine the value of the constant a. (2 marks)

$$0.1 + 5a + 0.25 + 0.2 + 3a + a = 1$$
$$a = 0.05$$

(ii) Determine $E(X)$ and $Var(X)$. (2 marks)

$$E(X) = 4.4$$

$$Var(X) = 7.44$$

(iii) State the values of $E(1 - 2X)$ and $Var(1 - 2X)$. (2 marks)

$$E(1 - 2X) = 1 - 2 \times 4.4 = -7.8$$

$$Var(1 - 2X) = (-2)^2 \times 7.44 = 29.76$$

(b) A fair eight-sided dice has faces marked with the numbers 1, 2, 3, 3, 3, 4, 5 and 5.

Determine $E(Y)$, where Y is the score when the die is rolled once. (3 marks)

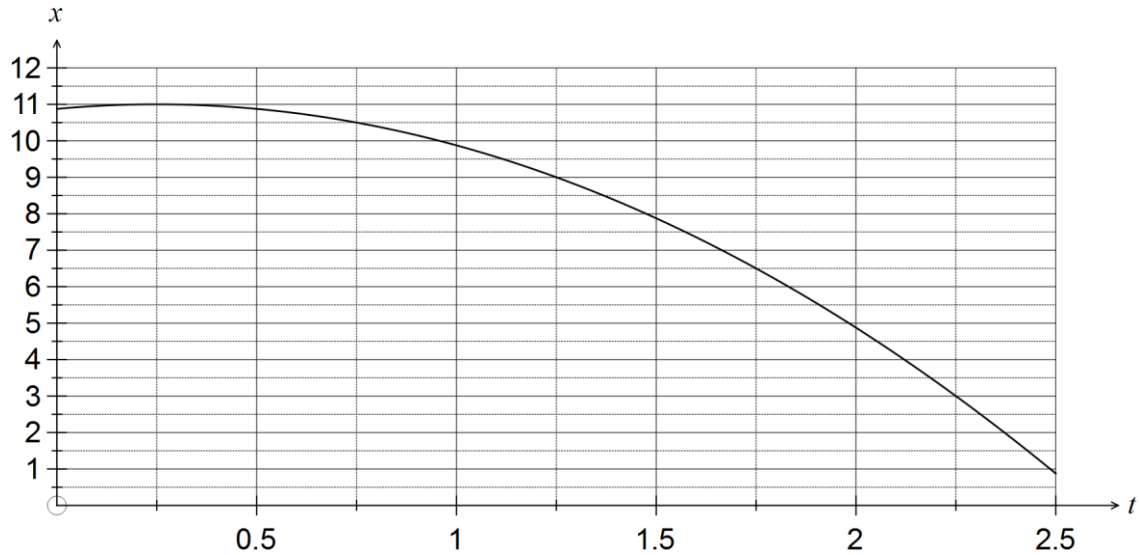| y	1	2	3	4	5
$P(Y = y)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{2}{8}$

$$E(Y) = \frac{1}{8} + \frac{2}{8} + \frac{9}{8} + \frac{4}{8} + \frac{10}{8} = \frac{26}{8} = 3.25$$

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Question 11   **(6 marks)**

A small toy train is able to travel along a straight track built on level ground.

The displacement, x in metres, of the train relative to a fixed point, is shown on the graph below for the interval $0 \leq t \leq 2.5$ seconds.

(a)   Use the graph to complete the table below.   (1 mark)

t	0.25	0.75	1.25	1.75	2.25
Displacement, x (m)	11.0	**10.5**	**9.0**	**6.5**	**3**

The area under the graph gives the total distance travelled by the train.

(b)   Estimate the area between the time axis and the displacement graph for the interval
        $0 \leq t \leq 2.5$ seconds using five **centred** rectangles of equal widths, and hence state the
        distance travelled by the train in this time.   (3 marks)

$$A = 0.5 \times (11 + 10.5 + 9 + 6.5 + 3)$$
$$= 0.5 \times 40$$
$$= 20$$

Train has travelled 20 m.

(c)   State, with reasons, whether your estimate in (b) is larger or smaller than the actual
        distance travelled by the train.   (2 marks)

Larger - each rectangle is slightly over-estimating
the area as the curve is concave down.

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Question 12   **(7 marks)**

In a random sample of 355 passenger cars in a city, it was found that 126 had a manual transmission.

(a)   Estimate the percentage of all passenger cars in the city with a manual transmission.

                                                                                         (1 marks)

$$126 \div 355 \approx 0.355$$

$$35.5\%$$

(b)   Calculate a 90% confidence interval for the estimate in (a), and briefly explain the meaning of the 90% level.   (3 marks)

$$0.355 \pm 1.645 \sqrt{\frac{0.355 \times (1 - 0.355)}{355}} \approx (0.313, 0.397)$$

90%: Such a confidence interval is expected to contain the true population proportion 90% of the time.

(c)   Assuming the estimate in (a) is correct, what size sample of passenger cars, to the nearest 10, would have to be taken in order for the width of a 98% confidence interval to be no more than 0.1?   (3 marks)

$$z_{0.98} = 2.326$$

$$n = \frac{2.326^2 \times 0.355(1 - 0.355)}{0.05^2}$$
$$= 495.7$$

$$\approx 500 \text{ to nearest 10 cars}$$

Question 13 **(8 marks)**

A telephone sales company have a success rate of 16% when calling customers selected at random from their large database.

(a) If a salesperson starts calling customers, determine the probability

(i) that their first success will be obtained on the fifth call they make. (2 marks)

$$P = (1 - 0.16)^4 \times 0.16$$
$$= 0.0797$$

(ii) they will make at least one sale in the first five calls. (2 marks)

$$S \sim B(5, 0.16)$$

$$P(S \geq 1) = 0.5818$$

(b) Calculate the number of calls a salesperson must make daily to achieve an average of 12 sales per day. (2 marks)

$$\bar{x} = np$$
$$n = \frac{12}{0.16} = 75$$

Should make 75 calls per shift.

(c) Determine how many calls a salesperson must make in a day to be 99% certain of achieving at least one sale in that time. (2 marks)

Consider making no sales n times:

$$(1 - 0.16)^n < 1 - 0.99 \implies n > 26.41$$

Make 27 calls.

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Question 14 **(7 marks)**

Analysis of the fuel consumption rate reported by a large number of owners of a particular model of car was observed to be normally distributed with a mean and standard deviation of 9.25 and 1.15 litres per 100 km respectively. The manufacturers claim that this type of car has a fuel consumption of 8.2 litres per 100 km.

(a) What percentage of these cars have a fuel consumption within one litre per 100 km of the manufacturers claim? (2 marks)

$$P(7.2 < X < 9.2) = 0.4453$$
$$\approx 44.5\%$$

(b) 15% of cars have fuel consumption of more than k litres per 100 km. Determine the value of k. (1 mark)

$$k = 10.44 \text{ litres per 100 km.}$$

(c) The fuel gauge of a randomly selected car shows 13.6 litres of fuel remain in the tank. Determine the probability that the car will reach its destination, 170 km away, without running out of fuel. (2 marks)

$$13.6 \div 1.7 = 8 \text{ litres per 100 km}$$

$$P(X < 8) = 0.1385$$

(d) An independent auditor found that the consumption figures supplied by the car owners had been incorrectly calculated. In order to correct each figure, it was necessary to use the formula $y = 0.95(x - 0.45)$, where x is the original figure and y is the corrected figure.

Determine the mean and standard deviation of the normal distribution of the corrected figures. (2 marks)

$$\bar{y} = 0.95(9.25 - 0.45) = 8.36$$

$$\text{sd}_Y = 1.15 \times 0.95 = 1.09$$

(Units L/100 km)

Question 15 **(7 marks)**

(a) The continuous random variable X has probability density function

$$f(x) = \begin{cases} ax + b & 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$. It is also know that $P(X < 2) = \dfrac{5}{8}$.

Determine the values of the constants a and b. (4 marks)

$$\int_1^3 \boldsymbol{ax+b}\,\mathrm{dx}=1$$

$$4{\cdot}a+2{\cdot}b=1$$

$$\int_1^2 \boldsymbol{ax+b}\,\mathrm{dx}=5/8$$

$$\frac{3{\cdot}a}{2}+b=\frac{5}{8}$$

$$\begin{bmatrix} 4{\cdot}a+2{\cdot}b=1 \\ \frac{3{\cdot}a}{2}+b=\frac{5}{8} \end{bmatrix}\Big|\,a,b$$

$$\left\{a=-\frac{1}{4},\,b=1\right\}$$

(b) Another continuous random variable Y has probability density function

$$g(y) = \begin{cases} \dfrac{6-y}{18} & 0 \le y \le 6 \\ 0 & \text{elsewhere} \end{cases}$$.

Determine the mean and standard deviation of Y. (3 marks)

$$\int_0^6 y\times\frac{6-y}{18}\,\mathrm{dy}$$

$$2$$

$$\int_0^6 \frac{6-y}{18}(y-2)^2\,\mathrm{dy}$$

$$2$$

$$\sqrt{\text{ans}}$$

$$\sqrt{2}$$

Mean is 2 and sd is $\sqrt{2}$.

Question 16   **(7 marks)**

The function f is defined as $f(x) = \ln(x^2 + e^2) - 1$.

(a)   Show that the equation of the tangent to $y = f(x)$ when $x = e$ is $y = \dfrac{x}{e} + \ln 2$.   (4 marks)

$$f'(x) = \frac{2x}{x^2 + e^2}$$

$$f'(e) = \frac{2e}{e^2 + e^2} = \frac{1}{e}$$

$$\begin{aligned} f(e) &= \ln(2e^2) - 1 \\ &= \ln 2 + 2\ln e - 1 \\ &= \ln 2 + 1 \end{aligned}$$

$$y - (\ln 2 + 1) = \frac{1}{e}(x - e) \;\Rightarrow\; y = \frac{x}{e} + \ln 2$$

(b)   Use the increments formula $\delta y \approx \dfrac{dy}{dx} \times \delta x$ to estimate the change in f when x increases

from 2 to 2.01, giving your answer rounded to four decimal places.   (3 marks)

$$x = 2; \;\; \delta x = 0.01$$

$$\delta y \approx \frac{2x}{x^2 + e^2} \times \delta x$$

$$\begin{aligned} &\approx \frac{2(2)}{2^2 + e^2} \times 0.01 \\ &\approx 0.0035121... \\ &\approx 0.0035 \text{ (4dp)} \end{aligned}$$

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Question 17   (8 marks)

(a)   The sound level, L decibels (dB), for a single sound of pressure p, in millipascals (mPa),

is calculated using the formula $L = 20\log\left(\dfrac{p}{0.02}\right)$, $p > 0$.

(i)   Determine the sound level corresponding to a sound pressure of 55 mPa.   (1 mark)

$$L = 20\log\dfrac{55}{0.02} = 68.8 \text{ dB}$$

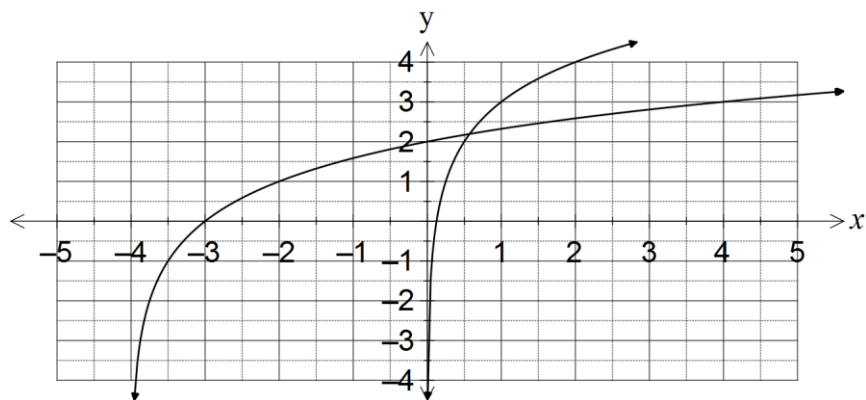(ii)   Determine the sound pressure corresponding to a sound level of 80 dB.   (1 mark)

$$80 = 20\log\dfrac{p}{0.02} \implies p = 200$$

(iii)   Calculate how many times greater the sound pressure is for a level of 140 dB than one of 80 dB.   (2 marks)

$$140 = 20\log\dfrac{p}{0.02} \implies p = 200000$$

$200000 \div 200 = 1000$ times greater

(b)   The graphs shown below are for $f(x) = \log_2(x) + a$ and $g(x) = \log_2(x + b)$.

(i)   State the values of the integer constants a and b.   (2 marks)
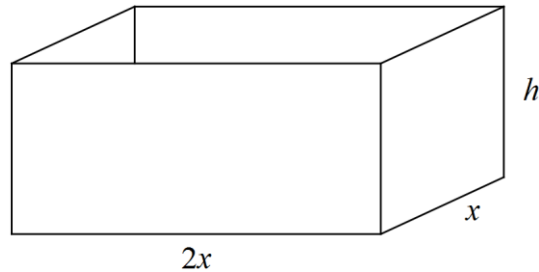
$a = 3, \quad b = 4$

(ii)   Solve $f(x) = g(x)$, correct to four decimal places.   (2 marks)

$\log_2(x) + 3 = \log_2(x + 4) \implies x = 0.5714$ *(Using CAS)*

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Question 18 **(7 marks)**

A budget of $225 is available to buy 4 mm thick steel sheeting to construct an open water tank in the shape of a rectangular prism of height h cm, that is twice as long ($2x$ cm) as it is wide (x cm). Cut to size, the sheeting costs 1.5 cents per square centimetre.

(a)   Show that the total cost of the steel, in dollars, is given by $\dfrac{3x^2 + 9xh}{100}$.   (1 mark)

$$C = \frac{1.5\big((x)(2x) + (x + 2x + x + 2x)(h)\big)}{100}$$
$$= \frac{3x^2 + 9xh}{100}$$

(b)   Assuming the whole budgeted amount is used to buy steel sheeting, show that volume of the tank in cubic centimetres is given by $V = 5000x - \frac{2}{3}x^3$.   (2 marks)

$$\frac{3x^2 + 9xh}{100} = 225 \implies h = \frac{22500 - 3x^2}{9x}$$

$$V = LWH$$
$$= (x)(2x)\left(\frac{22500 - 3x^2}{9x}\right)$$
$$= 5000x - \frac{2}{3}x^3$$

See next page

(c) Use calculus methods to determine the dimensions of the water tank that maximises the volume, and state this volume. (4 marks)

$$\frac{dV}{dx} = 5000 - 2x^2$$

$$= 0 \quad \text{when} \quad x = -50, x = 50$$

$$x = 50 \text{ cm}$$

$$2x = 100 \text{ cm}$$

$$h = \frac{22500 - 3(50)^2}{9(50)} = \frac{100}{3} \text{ cm}$$

Dimensions are 50 by 100 by $33\frac{1}{3}$ cm

Volume $= (50)(100)(\frac{100}{3}) = \frac{500000}{3} \approx 166667 \text{ cm}^3$

Question 19   **(9 marks)**

(a) Which of the following statistics would be easiest to collect? Justify your response.

(2 marks)

     A: The proportion of school students who have a part-time job.

     B: The proportion of part-time workers who are school students.

> A
>
> Much easier to obtain a sample of school students than part-time workers, as part-time workers spread across many varied occupations.

(b) Briefly define a random sample.   (1 mark)

> A random sample is one in which every member of the population has an equal chance of being selected.

(c) A random sample of Year 12 students was undertaken from which the 90% confidence interval for the proportion of students who planned to take a gap year after finishing Year 12 was determined to be (0.034, 0.252).

     (i) Explain why the reliability of this interval estimate is not immediately evident.

(2 marks)

> The sample size is not stated, and unless the sample size is large, the distribution of sample proportions is unlikely to be normal, leading to an unreliable interval.
>
> *(NB It can be determined from interval that n=28, but this is not immediately evident.)*

     (ii) Determine the corresponding 95% confidence interval based on this survey.

(4 marks)

> $$p = \frac{0.034 + 0.252}{2} = 0.143$$
>
> $$E = 0.252 - 0.143 = 0.109$$
>
> $$E_{NEW} = \frac{1.96}{1.645} \times E = \frac{1.96}{1.645} \times 0.109 = 0.130$$
>
> $$0.143 \pm 0.130 = (0.013,\ 0.273)$$

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Question 20 (9 marks)

The points A, B, C, D, E, F, G and H lie on the graph of the continuous function $y = f(x)$.
The table below contains information about the sign of $f(x)$, $f'(x)$ and $f''(x)$ at these points.

Point	A	B	C	D	E	F	G	H
x	-2	-1	0	2	4	6	7	8
$f(x)$	+	0	+	+	+	+	0	-
$f'(x)$	-	0	+	0	-	-	0	-
$f''(x)$	+	+	0	-	0	+	0	-

There are no other points at which $f(x)$, $f'(x)$ and $f''(x)$ are equal to zero.

(a)   For the graph of this function, state all points that are

(i)   roots. (1 mark)
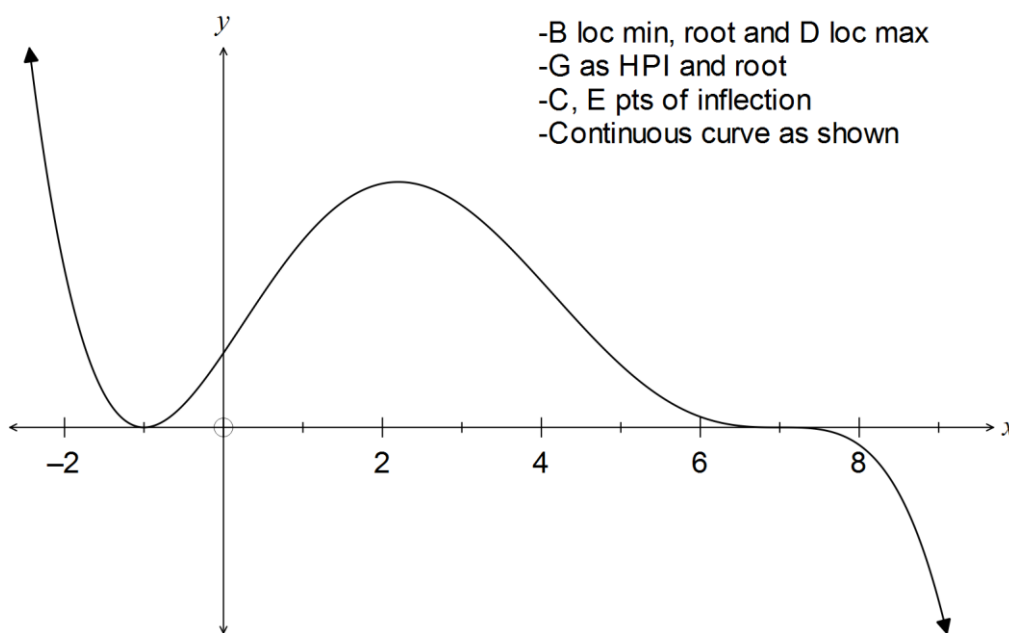
B and G

(ii)   points of inflection but not stationary points. (1 mark)

C and E

(b)   Describe the nature of the graph of this function at point

(i)   B. (1 mark)

Local minimum

(ii)   G. (2 marks)
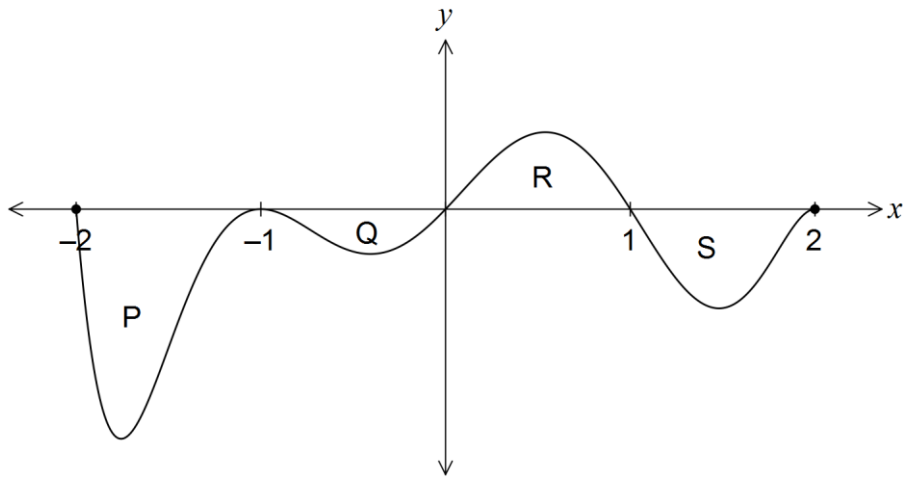
Horizontal point of inflection

(c)   Sketch a possible graph of $y = f(x)$ on the axes below. (4 marks)

-B loc min, root and D loc max
-G as HPI and root
-C, E pts of inflection
-Continuous curve as shown

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Question 21   **(8 marks)**

The graph of the function $y = f(x)$ is shown below over the domain $-2 \le x \le 2$.

The areas of regions P, Q, R and S enclosed by the curve and the x-axis are 5, 1, 2 and 3 square units respectively.

(a) Determine the area enclosed by the curve and the x-axis for $-1 \le x \le 1$.   (1 mark)

$$1 + 2 = 3 \text{ sq units}$$

(b) Determine the value of

(i) $\displaystyle\int_{-2}^{2} f(x) \, dx$.   (2 marks)

$$-5 + (-1) + 2 + (-3) = -7$$

(ii) $\displaystyle\int_{-1}^{1} 2f(x) \, dx + \int_{1}^{2} 3f(x) \, dx$.   (2 marks)

$$\int_{-1}^{1} 2f(x) \, dx + \int_{1}^{2} 3f(x) \, dx = 2(2-1) + 3(-3)$$
$$= 2 - 9$$
$$= -7$$

(iii) $\displaystyle\int_{-2}^{1} 2x + f(x) \, dx$.   (3 marks)

$$\int_{-2}^{1} 2x + f(x) \, dx = \int_{-2}^{1} 2x \, dx + \int_{-2}^{1} f(x) \, dx$$
$$= \left[x^2 \right]_{-2}^{1} + (-5) + (-1) + 2$$
$$= 1 - (-2)^2 - 4$$
$$= -7$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____